



# Mark Scheme (Results)

Summer 2023

Pearson Edexcel International Advanced Level  
In Decision Mathematics (WDM11)  
Paper 01

| Question Number | Scheme  | Marks                       |
|-----------------|---|-----------------------------|
| 1. (a)          |   | M1<br>A1<br>M1<br>A1<br>(4) |
| (b)             | Activity H can be delayed by $23 - 9 - 4 = 10$ (days)                               | B1ft<br>(1)                 |
| (c)             | Lower bound = $\frac{5+7+6+\dots+6+5}{33} = \frac{79}{33} = 2.393\dots = 3$ workers | M1 A1<br>(2)                |
| (d)             | <p>e.g.</p>   | M1 A1<br>A1<br>(3)          |
|                 |   | 10 marks                    |

### Notes for Question 1

**a1M1:** All top boxes complete, values generally increasing in the direction of the arrows (so generally going from ‘left to right’ across the network), condone one ‘rogue’ value (if values do not increase in the direction of the arrows then if one value is ignored and the remaining values do increase in the direction of the arrows then this is considered to be a single rogue value). Note that all values in the top boxes could be incorrect but it can still score the M mark if the values are **increasing** in the way stated above

**a1A1:** CAO – **all** values correct in the top boxes

**a2M1:** All bottom boxes complete (but condone a blank box for the late event time at the end event node and/or no zero value for the late event time at the start event node for the **M** mark only). Values generally decreasing in the opposite direction of the arrows (so generally going from ‘right to left’ across the network), condone one ‘rogue’ (as described above in **a1M1**)

**a2A1:** CAO – **all** values correct in the bottom boxes

**b1B1ft:** Correct calculation seen for their H (provided total float is non-negative). Correct answer or the correct answer following through the event times for H with no working seen scores **B0** – must see all three numbers in their calculation (e.g.  $23 - 9 - 4 = 10$ , or  $9 + 4 = 13$ ,  $23 - 13 = 10$ , etc.)

**c1M1:** Attempt to find lower bound: (a value in the interval  $[71 - 87]$  / their finish time) **or** (sum of all the activities / their finish time) **or** (as a minimum) an awrt 2.4

**c1A1:** CSO – requires the correct answer of 3 **and** either a **correct** calculation **or** awrt 2.4. An answer of 3 with no working scores no marks in this part

**d1M1:** Not a cascade chart. 4 workers used at most, at least 10 activities placed

**d1A1:** 4 workers. All 16 activities present just once. Condone at most **two** errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA

**d2A1:** CAO – no errors, all 16 activities present just once.

| Activity | Duration | Time interval | IPA        |
|----------|----------|---------------|------------|
| A        | 5        | 0 – 11        | -          |
| B        | 7        | 0 – 7         | -          |
| C        | 6        | 0 – 16        | -          |
| D        | 4        | 7 – 11        | B          |
| E        | 5        | 11 – 16       | A, D       |
| F        | 6        | 7 – 16        | B          |
| G        | 2        | 7 – 16        | B          |
| H        | 4        | 9 – 23        | C, G       |
| I        | 2        | 16 – 30       | C, E, F, G |
| J        | 7        | 16 – 23       | C, E, F, G |
| K        | 5        | 23 – 30       | H, J       |
| L        | 4        | 23 – 27       | H, J       |
| M        | 8        | 23 – 33       | H, J       |
| N        | 3        | 28 – 33       | I, K       |
| P        | 6        | 27 – 33       | L          |
| Q        | 5        | 27 – 33       | L          |

| Question Number | Scheme   | Marks                        |
|-----------------|--|------------------------------|
| <b>2.(a)</b>    | The 13 was used as a pivot in the first pass as all numbers greater than 13 are on the left (of the 13) and all numbers less than 13 are on the right (of the 13) (and this is not the case for any other value)                     | <b>B1 (1)</b>                |
| <b>(b)</b>      | 17 33 14 25 23 28 21 13 9 6 10<br>33 17 25 23 28 21 14 13 9 10 <b>6</b><br>33 25 23 28 21 17 14 13 10 <b>9 6</b><br>33 25 28 23 21 17 14 13 <b>10 9 6</b><br>33 28 25 23 21 17 14 <b>13 10 9 6</b><br>33 28 25 23 21 17 14 13 10 9 6 | M1<br>A1<br>A1<br><b>(3)</b> |
| <b>(c)</b>      | Bin 1: <b>33 28 23</b><br>Bin 2: <b>25 21</b> 17 14 6<br>Bin 3: 13 10 9  | M1<br>A1<br><b>(2)</b>       |
|                 |  | <b>6 marks</b>               |

### Notes for Question 2

**a1B1:** pivot value of 13 correctly stated + correct reasoning (give bod provided there is a clear intention of ‘values greater than the pivot, pivot, values less than the pivot’ e.g. ‘after the first pass all the values greater than 13 are to the left and all values smaller are on the right’. Allow for **B1** a statement such as, ‘values on one side are bigger than 13 and on the other are smaller’, however just a statement like ‘all the values greater than 13 are to the left’ is **B0** – must have an indication of values on both sides of the 13)

**PLEASE NOTE NO MISREADS IN PARTS (b) AND (c) – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE IN (b)**

**b1M1:** Bubble sort. Consistent direction, end number (6) in place and the list beginning with the correct first four numbers (33 17 25 23) after the first pass. Do check these carefully as some candidates show the result of each comparison and swap in their first pass. Consider the placement of the candidate’s numbers, rather than what the candidate labels each line of their pass. For example, assume that the first time that the 6 appears at the end of the list is the end of their first pass. Their first pass must be of the form 33 17 25 23 x 6 where x is either 5, 6 or 7 numbers

**b1A1:** The first, second **and** third passes correct

**b2A1:** Fourth and fifth passes correct – **must include a fifth pass** (ISW after correctly completing the fifth pass). If after the fourth pass they state that the list is in order and simply re-write the list then **A0** (but give bod if it could be interpreted as a fifth pass)

**SC in (b):** Ascending sort: First two passes correct scores **M1** only in (b)

17 14 25 23 28 21 13 9 6 10 33 followed by 14 17 23 25 21 13 9 6 10 28 33

**c1M1:** The **correct** first five values placed correctly (so must be the 33 28 25 23 and the 21) and at least eight values placed in bins - condone cumulative totals for **M1** only (the bold values)

**c1A1:** CSO – no additional or repeated values

| Question Number           | Scheme   |       |     |      |     | Marks           |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
|---------------------------|--|-------|-----|------|-----|-----------------|-----|-----|-----|-----|------|-------|-----|------|---|-----|------|----|-----|---|----|-----|---|----|---|---|-----|---|---|---|---|--|--|--|--|----|
| 3.(a)                     | <table><tr><td><math>N</math></td><td><math>A</math></td><td><math>B</math></td><td><math>C</math></td><td><math>D</math></td></tr><tr><td>4217</td><td>421.7</td><td>421</td><td>4210</td><td>7</td></tr><tr><td>421</td><td>42.1</td><td>42</td><td>420</td><td>1</td></tr><tr><td>42</td><td>4.2</td><td>4</td><td>40</td><td>2</td></tr><tr><td>4</td><td>0.4</td><td>0</td><td>0</td><td>4</td></tr><tr><td>0</td><td></td><td></td><td></td><td></td></tr></table> |       |     |      |     | $N$             | $A$ | $B$ | $C$ | $D$ | 4217 | 421.7 | 421 | 4210 | 7 | 421 | 42.1 | 42 | 420 | 1 | 42 | 4.2 | 4 | 40 | 2 | 4 | 0.4 | 0 | 0 | 4 | 0 |  |  |  |  | M1 |
|                           | $N$  | $A$   | $B$ | $C$  | $D$ |                 |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
|                           | 4217   | 421.7 | 421 | 4210 | 7   |                 |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
|                           | 421  | 42.1  | 42  | 420  | 1   |                 |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
|                           | 42   | 4.2   | 4   | 40   | 2   |                 |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
|                           | 4  | 0.4   | 0   | 0    | 4   |                 |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
|                           | 0  |       |     |      |     |                 |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
| Output values: 7, 1, 2, 4 |  |       |     |      | A1  |                 |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
|                           |  |       |     |      | A1  |                 |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
|                           |  |       |     |      | A1  |                 |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
|                           |  |       |     |      | A1  |                 |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
|                           |  |       |     |      | (4) |                 |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
| (b)                       | The first value is the units digit of $N$ , the second value is the tens digit, the third is the hundreds digit, and so on   |       |     |      |     | B2, 1, 0<br>(2) |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |
|                           |  |       |     |      |     | 6 marks         |     |     |     |     |      |       |     |      |   |     |      |    |     |   |    |     |   |    |   |   |     |   |   |   |   |  |  |  |  |    |

### Notes for Question 3

**a1M1:** At least three rows of cells completed with a correct first row – condone repeated values in all columns or a single value in each row

**a1A1:** CAO – the values in the second and third row correct

**a2A1:** CAO – fourth row correct and a zero only in the fifth  $N$  row

**a3A1:** Correct outputs (7, 1, 2, 4) – dependent on the first **four** rows being correct – the output must either be stated on the given answer line or ‘output 7, 1, 2, 4’ must be clearly written somewhere near the table (do not bod column D being circled, etc.). Condone the output being stated as 7124

**b1B1:** Indication that the outputs are the digits of  $N$

**b2B1:** Indication that the digits are in the reverse order

Examples of **B1 B1**:

- The output is  $N$  (**or** the original input **or** 4217) in reverse order
- The output is  $N$  backwards
- The output is (the digits of)  $N$  written right to left
- The first value is the unit digit of  $N$ , the second value is the tens digit, and so on

Examples for **B1 B0**:

- The output is each number of  $N$
- The output is  $N$
- Output values are the values that make up the original input

Examples for **B0 B0**:

- The output comes from/is derived from the original input
- The algorithm removes the last digit step by step
- The output values are the last digits of  $N$
- The output is the last digit of  $N$
- The output is  $N$  + contradictory statement (e.g. ‘output is  $N$  or a number that is smaller than  $N$ ’)

Example for **B0 B1** (not common):

- The output is in reverse/right to left/backwards (so no mention of  $N$  **or** original input **or** 4217)

| Question Number | Scheme  | Marks           |
|-----------------|---|-----------------|
| <b>4.(a)</b>    | $y \leq 2x$ and $3x + y \leq 30$  | B1              |
|                 | Correct method for finding the third boundary of the feasible region  | M1              |
|                 | $5y \geq 6x + 10$   | A1 (3)          |
| <b>(b)</b>      | (6, 12)   | B1              |
|                 | Solving simultaneous equations to find the other two vertices   | M1              |
|                 | $\left(\frac{5}{2}, 5\right)$ and $\left(\frac{20}{3}, 10\right)$   | A1 (3)          |
| <b>(c)</b>      | $\left(\frac{5}{2}, 5\right) \rightarrow P = 20$<br>$\left(\frac{20}{3}, 10\right) \rightarrow P = \frac{130}{3}$<br>$(6, 12) \rightarrow P = 48$ | M1<br>A1        |
|                 | Optimal vertex is (6,12) with $P = 48$  | A1 (3)          |
| <b>(d)</b>      | $12 + 12k \square 2\left(\frac{20}{3}\right) + 10k$ where $\square$ is any inequality or equals   | M1              |
|                 | $k \geq \frac{2}{3}$  | A1 (2)          |
|                 |   | <b>11 marks</b> |

#### Notes for Question 4

**a1B1:** Both correct inequalities:  $y \leq 2x$  and  $3x + y \leq 30$  (or equivalent if they have been rearranged)

**a1M1:** Correct equation for the third boundary of the feasible region, e.g.  $\frac{y-20}{14-20} = \frac{x-15}{10-15}$  would score

**M1** (so does not need to be simplified). Condone any inequality sign used (provided that if the inequality sign was replaced with an 'equals' then the equation ( $5y = 6x + 10$ ) would be correct). ISW once the correct equation has been seen (so condone the correct unsimplified equation if not simplified correctly)

**a1A1:** CAO for third inequality – must be three terms only but need not be further simplified (e.g.  $12x - 10y + 20 \leq 0$ ,  $y \geq 1.2x + 2$  etc. scores **A1**)

**b1B1:** (6, 12) or  $x = 6$ ,  $y = 12$

**b1M1:** Correct method for solving their simultaneous equations (so  $y = 2x$  with their  $5y = 6x + 10$  or  $3x + y = 30$  with their  $5y = 6x + 10$ ) to find both the  $x$  and  $y$ -coordinates of at least one of the other two vertices (can be implied by either vertex stated correctly – condone non-exact values (to at least 3 sf) for this mark if no working seen)

**b1A1:** CAO (2.5, 5) and  $\left(\frac{20}{3}, 10\right)$  (must be exact, so accept  $(6\frac{2}{3}, 10)$ ,  $(6.\dot{6}, 10)$ , etc. but not  $(6.6666\dots, 10)$ )

**c1M1:** Testing their three **vertices** (not just points in the FR) in the correct objective function ( $P = 2x + 3y$ ). Condone one slip only when applying the objective function to one of their three vertices (so they must apply the correct objective to at least two of their three points but condone a single slip in the third)

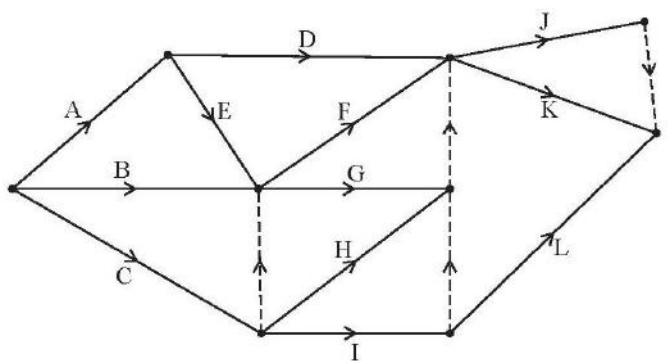
**c1A1:** At least two correct (therefore exact) points tested correctly (so at least two correct values of  $P$  explicitly stated – allow awrt 43.3 when testing  $(20/3, 10)$ )

**c2A1:** CSO all three correct (therefore exact) points tested correctly (so all three correct values of  $P$  explicitly seen – allow awrt 43.3 when testing  $(20/3, 10)$ ) with a clear indication of which is the optimal vertex (this can be achieved by either making it clear that (6, 12) is the optimal point or that  $P = 48$  is the maximum (as one implies the other))

**d1M1:** (6, 12) **and** their  $\left(\frac{20}{3}, 10\right)$  (does not need to be exact) correctly substituted into  $P = 2x + ky$  and compared (by comparing we mean forming an equation or any inequality). **Or** comparing  $-\frac{2}{k}$  with  $-3$  (so using the gradient of the objective line) **or** comparing  $\frac{2}{k}$  with  $3$  **or** their reciprocals (e.g.  $\frac{k}{2}$  compared with  $\frac{1}{3}$  etc.) but comparing  $-\frac{2}{k}$  with  $3$  is **M0** (by comparing we mean forming an equation or any inequality with the **correct** pairs of values)

**d1A1:** CAO either  $k \geq \frac{2}{3}$  or  $k > \frac{2}{3}$  **only** (or exact equivalents) – if any other answers given with  $k \geq \frac{2}{3}$  e.g.  $k \geq -1$  then **A0** (unless clearly rejected). Correct answer with no working can score both marks in this part. Please ensure that if working is shown that the correct answer of  $k \geq \frac{2}{3}$  comes from correct working e.g.

$$-\frac{2}{k} \geq -3$$

| Question Number | Scheme   | Marks                             |
|-----------------|--|-----------------------------------|
| 5. (a)          | e.g.            | M1<br>A1<br>A1<br>A1<br>A1<br>(5) |
| (b)             | Duration of activity K is $33 - 10 - 7 - 8 = 8$ <b>or</b> the path AEFJ has a duration of $< 33$ | M1                                |
|                 | Therefore the duration of J is less than 8 hours or $0 < \text{dur}(J) < 8$                      | A1<br>(2)                         |
|                 |  | <b>7 marks</b>                    |

### Notes for Question 5

Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finish at the correct event and **appears only once in the network**, e.g. 'G dealt with correctly' requires the correct precedences for this activity, i.e. B, C and E labelled correctly and leading into the same node and G starting from that node but do not consider the end event for G **so use the table below for checking as there a number of acceptable answers. Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity E is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

**Ignore incorrect or lack of arrows on the activities for the first four marks only (but assume that they are in the 'correct' direction for checking purposes)**

**a1M1:** At least seven activities (labelled on arc), one start and at least two dummies placed

**a1A1:** Activities A, B, C, D and E dealt with correctly

**a2A1:** Activities F, G, H and I dealt with correctly (so a dummy is required at the end of C + correct arrow)

**a3A1:** Activities J, K and L dealt with correctly (so **at least** two further dummies required + correct arrows)

**a4A1:** CSO – all arrows correctly placed for each activity with one finish and at most four dummies.

**Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only.**

**Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned**

#### Extremely useful for checking (a)

| Activity   | A | B | C | D | E | F       | G       | H | I | J             | K             | L |
|------------|---|---|---|---|---|---------|---------|---|---|---------------|---------------|---|
| <b>IPA</b> | - | - | - | A | A | B, C, E | B, C, E | C | C | D, F, G, H, I | D, F, G, H, I | I |

**b1M1:** Either the correct method for calculating the duration of activity K seen (e.g.  $33 - 10 - 7 - 8$  or an answer of 8 can imply this mark) **or** stating the path AEFJ will have a duration  $< 33$  or  $\leq 33$

**b1A1:** Correct indication that the duration of K being 8 implies that J's duration is  $< 8$  (allow  $\leq 8$  but **A0** for  $\leq 7$ ). As a minimum for both marks candidates must either say that the duration of K is 8 (e.g.  $K = 8$ ) **or** that the path AEFJ has a duration  $< 33$  or  $\leq 33$ , **together** with the duration of J being either  $< 8$  or  $\leq 8$ . A lower limit is not required but if stated then it must be either  $> 0$  or  $\geq 0$  **only**



| Question Number  | Scheme  | Marks                                    |
|--|---|--|
| 6.(a)(i)   | <div><p>Shortest path from A to J: ADCFEHGJ</p></div>   | M1<br>A1 (ABDC)<br>A1 (FE)<br>A1ft (HGJ) |
| (a)(ii)  | Length of shortest path from A to J: 67 (miles)         | A1ft (6)                                 |
| (b)  | AC + EJ = A(D)C + E(HG)J = 21 + 31 = 52                 | M1 A1                                    |
|  | AE + CJ = A(DCF)E + C(FEHG)J = 36 + 46 = 82             |  |
|  | AJ + CE = A(DCFEHG)J + C(F)E = 67 + 15 = 82             | A1                                       |
|  | Route length is 315 + 52 = 367 (miles)                  | A1ft (4)                                 |
| (c)  | Pass through G a total of 3 times                       | B1 (1)                                   |
| (d)  | Difference in inspection routes is 67 – 52 = 15 (miles) | B1ft (1)                                 |
| (e)  | Arcs CF and EF do not need to be repeated               | B1 (1)                                   |
|  |   | 13 marks                                 |
| Notes for Question 6   |   |  |
| <p>In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 40 35 31 in that order (so 40 31 35 is incorrect)</p> <p>It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling</p> |   |  |

**a1M1:** A larger value replaced by a smaller value at least twice in the working values at either C, E, F, G or J

**a1A1:** All values at A, B, D and C correct and the working values in the correct order

**a2A1:** All values at F and E correct and the working values in the correct order

**a3A1ft:** All values in H, G and J correct on the follow through and the working values in the correct order.

To follow through G say check that the working value(s) at G follow from the candidate's final values for the nodes that are directly attached to G (which are D, F, H and J). For example, **if** correct then the order of labelling of nodes D, F and H are 3, 5 and 7 respectively so the working values at G should come from D, F and H in that order. The first working value at G should be their 23 (the Final value at D) + 34 (the weight of the arc DG), the second working value at G should be their 31 (the Final value at F) + 15 (the weight of the arc FG) and the third working value at G should be their 42 (the Final value at H) + 2 (the weight of arc GH). Repeat this exact process for H and J for the follow through for this mark

**a4A1:** Correct shortest path from A to J (ADCFEHGJ) **only** – not from J to A

**a5A1:** Follow through their final value at J **only** (condone lack of units) – if their answer is 67 but this is not their Final Value at J then **A0**

**Condone for the final two marks in (a) the 'shortest path' and 'length of shortest path' written on the wrong lines**

**b1M1:** Three distinct pairings of nodes A, C, E and J

**b1A1:** Any one row correct including pairing **and** total

**b2A1:** All three rows correct including pairings **and** totals

**b3A1ft:** Correct route length (367) from the correct pairing or follow through 315 + their least total from a choice of three

**c1B1:** CAO (3)

**d1B1ft:** Correct answer of 15 either from correct or no incorrect working **or** follow through (their Final value at J from (a) – their least repeat from (b)) – **this mark is dependent on having scored both M marks in (a) and (b)**

**e1B1:** CAO (CF, EF only)

| Question Number | Scheme   | Marks              |
|-----------------|--|--------------------|
| 7. (a)          | Prim: AH, FH, EH, FG, DG, CG, BC   | M1 A1<br>A1<br>(3) |
| (b)             | Initial upper bound $2(201) = 402$ (km)  | B1ft<br>(1)        |
| (c)             | A – H – F – G – D – B – C – E – A<br>$27 + 28 + 31 + 29 + 32 + 26 + 38 + 37 = 248$   | M1<br>A1<br>(2)    |
| (d)             | Nearest neighbour starting at E has a length of $212 + x$<br>As $x \leq 35 \Rightarrow$ the NN route starting at E is at most 247 (km) and therefore the NN starting at E gives the better upper bound as it is less than the one starting at A (which was 248 (km)) | M1 A1<br>(2)       |
| (e)             | Lower bound is given by $(201 - 27) + 27 + x = 235$  | M1                 |
|                 | $x = 34$   | A1                 |
|                 | $235 \leq \text{optimal length} \leq 246$  | M1 A1<br>(4)       |
|                 |  | <b>12 marks</b>    |

#### Notes for Question 7

**a1M1:** Prim's – first three arcs correctly chosen in order (AH, FH, EH, ...) **or** first four nodes {A, H, F, E, ...} correctly chosen in order. If any explicit rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, -, -, 4, 3, -, 2} so check there too. Starting at any other node apart from A can score M1 only for first three arcs chosen correctly

**a1A1:** First five arcs correctly chosen in order (AH, FH, EH, FG, DG, ...) **or** all eight nodes {A, H, F, E, G, D, C, B} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 8, 7, 6, 4, 3, 5, 2} (**no** missing numbers)

**a2A1:** CSO – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

**b1B1ft:** Follow through double the stated length of their MST

**c1M1:** NN starting at A – must have at least A – H – F – G – D – ... allow if stated in terms of arcs

**c2A1:** CAO on length (248) **and** route (must return to A but can be stated in terms of arcs)

**d1M1:** Calculating the correct length of the NN route starting at E ( $212 + x$ ) **and** attempting to use the range of values for  $x$  to determining the better upper bound (implied by 247 seen or [244, 247]).

**d1A1:** Correct best upper bound stated (the one starting at E) together with a correct comparison of **248** (possibly implicit - if this value is not explicitly stated in (d) then 248 must have been seen in (c)) with 247 (or an indication of 'at most' 247). **For those who obtained an answer of 211 in (c) and say that  $212 + x$  is always bigger (without using the given interval for  $x$  to find the UB) then no marks in this part**

**e1M1:** Correct method for calculating  $x$  (which is the weight of MST from (a)/(b) – 27 + two smallest arcs incident to A (the 27 and  $x$ ) equal to 235). If using the doubled value from (b) then **M0**. If not using the weight of the MST from (a) then they must be using either 174 **or**  $26 + 30 + 29 + 31 + 28 + 30$  **or** explicitly using the correct six arcs only (BC, CG, GD, GF, FH, HE) so not just circled in one of the tables. The correct value of  $x$  (with either no working or no incorrect working) clearly stated can imply this mark

**e1A1:** CAO for  $x$  (34) – **as a minimum** must have seen the calculation  $201 + x = 235$  to award this mark

**e2M1:** Any indication of an interval from 235 to either 246, 247 or 248 (this mark is **not** dependent on the previous M mark)

**e2A1:** CAO (condone  $235 < \text{optimal length} \leq 246$  and allow equivalent interval notation e.g.  $(235, 246]$  or  $[235, 246]$ ) – this mark is dependent on all previous marks in (e) (so must have found that  $x$  equals 34).

The correct interval (with no others) with no supporting working scores **M0A0M1A0**

The minimum requirement for full marks is:  $201 + x = 235 \Rightarrow x = 34 \therefore [235, 246]$

For those who simply state  $x = 34$  (only) followed by the correct interval they score **M1A0M1A0**

| Question Number | Scheme  | Marks           |
|-----------------|---|-----------------|
| <b>8.(a)</b>    | (Maximise) $x + y + z$  | B1              |
|                 | Subject to:   |                 |
|                 | $14x + 8y + 12z \leq 976$   | B1              |
|                 | $5z \geq 2x$  | B1              |
|                 | $\frac{1}{2}(x + y + z) \leq x (\Rightarrow -x + y + z \leq 0)$   | M1              |
|                 | $\frac{1}{5}(x + y + z) = y (\Rightarrow x - 4y + z = 0)$   | B1              |
|                 | Substituting $z = 4y - x$ into objective and constraints  | M1              |
|                 | Maximise ( $P =$ ) $5y$<br>Subject to:<br>$x + 28y \leq 488$<br>$7x - 20y \leq 0$<br>$2x - 5y \geq 0$<br>$x - 4y \leq 0$<br>$x \geq 0 \quad y \geq 0$ | A1 A1<br>(8)    |
| <b>(b)</b>      | Substituting $y = 16$ into constraints gives<br>$x \leq 40, x \leq \frac{320}{7}, x \geq 40, x \leq 64$   | M1              |
|                 | $\Rightarrow x = 40$ and therefore the maximum number of leadership prizes is 24  | A1<br>(2)       |
|                 |   | <b>10 marks</b> |

#### Notes for Question 8

**a1B1:** CAO ( $x + y + z$ ) – can be implied by seeing  $5y$  as the objective – do not penalise lack of ‘maximise’ here. Allow equal to any letter but not equal to a numerical value

**a2B1:** CAO ( $14x + 8y + 12z \leq 976$  or an unsimplified inequality equivalent to  $x + 28y \leq 488$ )

**a3B1:** CAO ( $5z \geq 2x$  (or equivalent) or an unsimplified inequality equivalent to  $7x - 20y \leq 0$ )

**a1M1:** Correct method - must see  $\frac{1}{2}(x + y + z) \bullet x$  where  $\bullet$  is any inequality or  $=$ . The bracket must be present or implied by later working – this mark is implied by the stating of a correct/incorrect inequality that would come from  $\frac{1}{2}(x + y + z) \bullet x$  (possibly with  $z = 4y - x$  substituted)

**a4B1:** CAO  $\frac{1}{5}(x + y + z) = y$  (allow any equivalent unsimplified/simplified form for this mark) – simply stating that e.g.  $x + z = 4y$  scores this mark

**a2M1:** Eliminating  $z$  using the correct equation  $z = 4y - x$  from all three non-trivial constraints and objective

**a1A1:** At least two simplified constraints correct + correct objective (in  $x/y$  only) – condone lack of ‘maximise’ for this mark - accept any equivalent forms for the constraints provided that coefficients are integers and only a single term in  $x$  and/or  $y$

**a2A1:** CAO – must include ‘maximise’ or ‘max’ (but not ‘maximum’) and all 6 constraints – accept any equivalent forms for the constraints provided that coefficients are integers and only a single term in  $x$  and/or  $y$

**Condone using  $a$  for  $x$ ,  $s$  for  $y$  and  $l$  for  $z$  – if any other letter used then send to review**

**Must be using inequalities in  $x$  and  $y$  only in (b) – if inequalities are still in terms of  $z$  then  $z$  must be eliminated using the correct equation (e.g.  $z = 64 - x$ ). Attempting part (b) with equations (rather than inequalities) scores no marks (even if the correct answer of 24 is seen).**

**b1M1:** Substituting  $y = 16$  into at least two correct constraints from the following four:  $x + 28y \leq 488$ ,  $7x - 20y \leq 0$ ,  $2x - 5y \geq 0$  and  $x - 4y \leq 0$

**b1A1:** Correctly obtaining  $x \leq 40$  from  $x + 28y \leq 488$  **and**  $x \leq \frac{320}{7}$  from  $7x - 20y \leq 0$  **and**  $x \geq 40$  from  $2x - 5y \geq 0$  (condone lack of  $x \leq 64$  from  $x - 4y \leq 0$ ) then stating/implying that  $x = 40$  and therefore  $z = 24$  (or equivalent in context e.g. 24 leadership prizes)